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$\sqrt{2px-y^2}=z'$ and $-\sqrt{2px-y^2}=z''$. The limits of y are $\sqrt{px-x^2}-y'$ and $-\sqrt{px-x^2}=y''$. The limits of x are o, and p.

$$\therefore\ V=\int_o^p\int_{y''}^{y'}\int_{z''}^{z'}dxdydz=2\int_o^p\int_{y''}^{y'}\sqrt{2px-y^2}\,dxdy,$$

$$=2\int_o^p\left(x\sqrt{p^2-x^2}+2px\sin^{-1}\sqrt{\frac{p-x}{2p}}\right)dx$$

$$=2\left[-\tfrac{1}{3}\sqrt{(p^2-x^2)^3}+px^2\sin^{-1}\sqrt{\frac{p-x}{2p}}-\tfrac{1}{4}px\sqrt{p^2-x^2}+\tfrac{1}{4}\ p^3\sin^{-1}\frac{x}{p}\right]_o^p$$

$$=2p^3\left(\frac{1}{3}+\frac{\pi}{8}\right)=\tfrac{1}{12}p^3(8+3\pi).$$

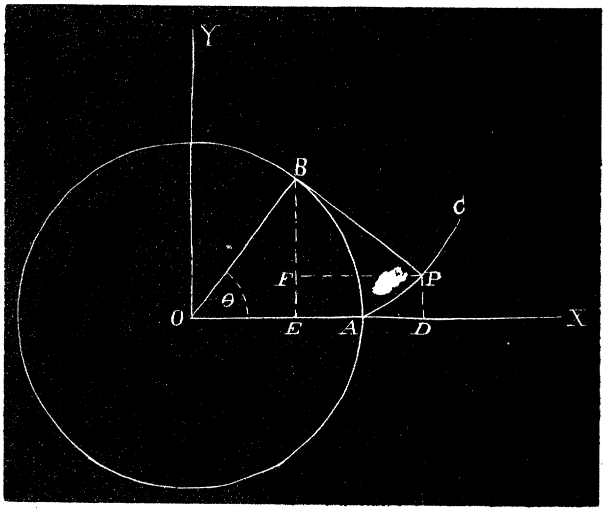
Also solved by *Professor Hume* and the *Proposer*.

6. Proposed by O. S. KIBLER, Superintendent of Schools, West Middleburg, Ohio.

A string is wound spirally twenty times around a cylinder 20 feet high and 2 feet in diameter. Through what distance will a dove fly in unwinding the string keeping it tense at all times (1) flying in the same plane and (2) not flying in the same plane?

Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University P. O., Mississippi.

Let the circle whose center is O and the curve AC be, respectively, the horizontal projections of the vertical cylinder and the dove's path.



Let P be any point of AC and A the original position of that end of the string to which the dove is attached.

Case I: THE DOVE FLYING IN ONE PLANE.

AC will be the involute of the circle. Denoting the angle AOB by θ and OA by a, $BA=a\theta=BP$.

The co-ordinates of P refered to axes as shown in the figure are,

$x=OD=OE+FP=a\cos\theta+a\theta\sin\theta$, $y=PD=BE-BF=a\sin\theta-a\theta\cos\theta$.

Therefore, $dx=(-a\sin\theta+a\theta\cos\theta+a\sin\theta)d\theta=a\theta\cos\theta d\theta$.

$dy=(a\cos\theta+a\theta\sin\theta-a\cos\theta)d\theta=a\theta\sin\theta\,d\theta$.

$ds^2=dx^2+dy^2=a^2\theta^2d\theta^2$; $ds=a\theta d\theta$; $s=\int a\theta d\theta=a\frac{\theta^2}{2}$, s being estimated from A.

$a=1$, and the θ-limits are 40π and 0. $\therefore$ $s=800\pi^2=7895$ feet, approximately.

Case II: THE STRING BEING KEPT HORIZONTAL.

BP, now, is the length of that part of the string unwound. Hence, BP is the hypotenuse of aright triangle whose base is the arc AB and whose altitude is the vertical distance of the dove from A. This may be represented by $c\theta$. It follows that $BP=\sqrt{a^2\theta^2+c^2\theta^2}=(a^2+c^2)^{\frac{1}{2}}\theta$.

The co-ordinates of P are:

$x=a\cos\theta+(a^2+c^2)^{\frac{1}{2}}\theta\sin\theta$,

$y=a\sin\theta-(a^2+c^2)^{\frac{1}{2}}\theta\cos\theta$,

$z=c\theta$, the axis of z coinciding with the cylinder's axis. $\therefore\ ds^2=dx^2+dy^2+dz^2=\Big[2a^2+2c^2-2a\ (a^2+c^2)^{\frac{1}{2}}+(a^2+c^2)\theta^2\Big]d\theta^2$.

$$\therefore\ s=\int\Big[2a^2+2c^2-2a(a^2+c^2)^{\frac{1}{2}}+(a^2+c^2)\theta^2\Big]^{\frac{1}{2}}d\theta=\sqrt{a^2+c^2}\left\{\frac{\theta}{2}\left(\frac{2a^2+2c^2-2a(a^2+c^2)^{\frac{1}{2}}}{a^2+c^2}+\theta^2\right)^{\frac{1}{2}}+\frac{a^2+c^2-a\sqrt{a^2+c^2}}{a^2+c^2}\log\left[\theta+\left(\frac{2a^2+2c^2-2a\sqrt{a^2+c^2}}{a^2+c^2}+\theta^2\right)^{\frac{1}{2}}\right]\right\}.$$

$a=1$, $c=\dfrac{1}{2\pi}$, and the θ-limits are 40π and 0.

$\therefore\ s=7995$ feet, approximately.

[*C. E. Myers* solves for case I. and gets 7895.517768 feet. *Prof. Zerr* solves by a different method and gets 8023.754 feet and 8124.910+feet, for his results.]

PROBLEMS.

12. Proposed by ISAAC L. BEVERAGE, Monterey, Virginia.

Given the equations $2z^3=x+3z$ and $5z^2=y+2z$. To find $\dfrac{dy}{dx}$ for $x=o$.

13. Proposed by J. A. CALDERHEAD, B. Sc., Superintendent of Schools, Limaville, Ohio.

A steamer whose course is due west and speed 10 knots is sighted by another steamer going at 8 knots; what course must the latter steer, so as to cross the track of the former at the least possible distance from her?

14. Proposed by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

Right triangles are inscribed in a circle whose centre $=(a,b)$, and radius$=c$. If one of the legs passes through a fixed point, prove that $c^2(x^2+y^2)=(a^2+b^2-c^2-ax-by)^2$ is the curve to which the other leg is always tangent; the fixed point being the origin of the co-ordinates.

15. Proposed by CHARLES E. MYERS, Canton, Ohio.

From a given quantity of material a cylindrical cup with circular bottom and open top is to be made, the cup to contain the greatest amount. What must be its dimensions?

Solutions to these problems should be received on or before May 1st.